

**Fermilab**

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Design Considerations for the  
Fermilab Test RFQ

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Introduction

A new concept in low energy proton/ion accelerators is being developed at accelerator facilities throughout the world. This concept is labeled the RFQ linac (Radio-Frequency Quadrupole) and was first proposed by Kapchinskii and Teplyakov<sup>1</sup> and more extensively developed by scientists at Los Alamos<sup>2,3</sup> where precise design procedures are being developed and a "proof of principle" experiment was performed. RFQ design and construction is currently being undertaken by groups at almost every major laboratory in the world (CERN, BNL, LASL, GSI, LBL, INS, Saclay, etc.).

We propose that Fermilab also participate in this development, to gain local knowledge in this new technology and also to improve the low energy portion of the Fermilab linac. In this note we review current progress in RFQ design concepts and outline design parameters for a Fermilab RFQ.

Our first RFQ is planned to be a 200 MHz structure accelerating  $H^-$  from ~30 keV (source energy) to 750 keV, and is a possible replacement for the Cockcroft-Walton, as well as a development project.

Outline of RFQ Design Constraints

The basic purpose of an RFQ is to accept a low energy injected beam and accelerate it to a higher energy (suitable for an Alvarez linac) with minimum phase space dilution and adequately large space charge acceptance. A basic procedure for RFQ optimization has been developed by Crandall, Stokes,

and Wangler<sup>3</sup> and is reviewed here in application to our Fermilab case.

The RFQ fields are obtained by excitation of a suitably shaped cavity. The quadrupole potential is:

$$U = \frac{V}{2} \left[ X \left( \frac{r}{a} \right)^2 \cos 2\psi + A I_0(kr) \cos kz \right] \cdot \sin(\omega t + \phi) \quad (1)$$

where  $V$  is the peak voltage difference between "vaness",  $a$  is the inter-vane radius (see Figure 1),  $I_0$  and  $I_1$  are Bessel functions,  $k = 2\pi/\beta\lambda$  with  $\beta$  the central particle velocity, and  $\lambda$ ,  $\omega$  are the excitation wavelength and frequency.  $(r, \psi, z)$  are the cylindrical coordinates and  $X, A$  are parameters determined by the longitudinal modulations of the cavity vanes, with  $m$  the modulation factor:

$$A = \frac{m^2 - 1}{m^2 I_0(ka) + I_0(mka)}$$

$$X = 1 - A I_0(ka) .$$

It can be verified that Equation (1) satisfies Maxwell's equations with appropriate boundary conditions.

The first term of (1) is an electric quadrupole field providing focusing in one transverse dimension and defocusing in the other. The time variation  $\sin(\omega t + \phi)$  provides alternate gradient focusing in both transverse dimensions.

The second term of (1) is radially defocusing and provides synchronous acceleration with longitudinal focusing within a separatrix.

In RFQ design the parameters of Equation (1) are varied within particular constraints for optimum acceptance. These constraints are:

(1) Maximum allowable field: The maximum field ( $\sim 1.5$  V/a) must not be greater than the sparking "breakdown" limit. This is usually expressed in

terms of the "Killpatrick limit"  $E_k$  with a criterion for maximum field

$$E_{\max} \lesssim 1.75 E_k.$$

For our 200 MHz Fermilab RFQ,  $E_k \cong 15$  MV/m and we require  $E_{\max} \lesssim 25$  MV/m.

Precise relationships between  $E_{\max}$  and  $V/a$  can be obtained using SUPERFISH<sup>4</sup>.

(2) Transverse focusing: Stability in radial focusing is required. The transverse focusing strength is characterized by the quantity

$$B = \frac{q\lambda^2}{Mc^2} \frac{XV}{a^2}.$$

In the smooth approximation the phase advance per AG focusing period is

$$\sigma_0 = \frac{B}{\sqrt{8\pi^2}}$$

$\sigma_0 < \pi$  is necessary for stable transport and  $0.5 \lesssim \sigma_0 \lesssim \pi/2$  is considered desirable. This implies

$$4.5 \lesssim B \lesssim 14.$$

In previous RFQ designs  $B$  was fixed at a value near 7. In our initial designs this will be followed, but a gradual variation of  $B$  is also possible.

It is important that the transverse focusing be strong enough for stable space charge transport. Studies of beam transport with space charge<sup>5</sup> have led to the following expression for the limiting current of an RFQ transport system (when  $B \lesssim 14$ )

$$I_T \cong \frac{.84}{6 \pi^2 Z_0} B \times V(\beta\gamma) |\phi_s|$$

where  $Z_0 = 377 \Omega$ ,  $\phi_s$  is the synchronous acceleration phase,  $\beta$ ,  $\gamma$  are the usual kinematic factors. (The above limit is further reduced by envelope

modulation and RF defocusing factors not included.)

(3) Longitudinal Stability: The RFQ must also have adequate stability in longitudinal motion. In an RFQ the longitudinal field is initially increased at a rate designed to minimize phase space dilution. Equations for longitudinal motion are:

$$\frac{d(W-W_s)}{ds} = \frac{\pi q AV}{2\beta_s \lambda} (\cos\phi - \cos\phi_s)$$

$$\frac{d(\phi-\phi_s)}{ds} \cong \frac{-2\pi}{Mc^2 \beta_s^3 \lambda} (W-W_s)$$

where  $\beta_s$ ,  $W_s$ ,  $\phi_s$  are the synchronous speed, kinetic energy and phase;  $\phi$  and  $W$  are an individual particle energy and phase and the above equations assume nonrelativistic motion and  $W$  is near  $W_s$ .

In the limit where  $A$ ,  $\beta_s$ ,  $\phi_s$  and  $V$  are constant the above equations describe phase oscillations in an RF bucket. Stable motion occurs within a "teardrop" shaped RF bucket (see figure 2) whose size is determined by the limits

$$\Delta W = \pm \sqrt{2q AV W_s |\sin\phi_s - \phi_s \cos\phi_s|}$$

and  $\Delta\phi$  total, the solution  $\phi$  of the equation:

$$|\tan \phi_s| = \frac{\phi - \sin\phi}{1 - \cos\phi}.$$

The beam must remain in the bucket defined by  $\Delta W$ ,  $\Delta\phi$  and  $\phi_s$ , and be confined with respect to space charge. Longitudinal stability sets a limit on current:<sup>5</sup>

$$I_L \lesssim \frac{\pi(.84) AV \phi_s^2 |\sin\phi_s|}{Z_o \lambda}.$$

Adiabatically establishing an RF bucket containing the beam is a necessary function

in RFQ design.

(4) Other constraints: Recent experience indicates that tuning sensitivity greatly increases with the length of the cavity, roughly as  $(L/\lambda)^2$ , where  $\lambda$  the RFQ wavelength. To avoid this problem we propose limiting our 200 MHz cavity to  $\approx 1.5$  m, 1 RF wavelength in total length.

Longitudinal focusing causes radial defocusing. The RF defocusing can be calculated from the parameter  $\Delta_{RF}$  given by

$$\Delta_{RF} = \frac{\pi^2 q VA}{4 W_s} \sin \phi_s .$$

One requires

$$|\Delta_{RF}| \ll B^2/8\pi^2$$

so that RF defocusing is small compared to the AG focusing.

#### Outline of RFQ Design

Los Alamos physicists have developed a fairly detailed procedure for RFQ design<sup>3</sup>, which we will follow in our initial approaches. The RFQ is conceptually split into four sections: a radial matching section, a "shaper", a "gentle buncher" and an accelerator section.

The variations of the aperture  $a$ , the modulation  $m$ , and the stable phase  $\phi_s$  along the RFQ are set by the requirements of these sections.

In the radial matching section, the transverse focusing strength is gradually increased from zero to full strength over 5-10 periods of  $\beta\lambda/2$ . This is done by decreasing  $a$  from some large value to a final value set by  $B$ :

$$a = \sqrt{\frac{q\lambda^2 V}{B mc^2}} .$$

The modulation  $m$  remains 1.0 and  $\phi_s$  is  $-90^\circ$  in this section. In the following "shaper" section,  $m$  is increased gradually from  $m = 1.00$  while  $\phi_s$  increases from  $-90^\circ$ . In Los Alamos designs

$$B = \frac{q\lambda^2}{Mc^2} \frac{XV}{a^2}$$

is usually kept constant. For our case the shaper should be  $\sim 0.5$  m long, and  $m$  increases to  $\sim 1.19$  and  $\phi_s$  to  $\sim -80^\circ$  following similar Los Alamos designs.

In the "Gentle Buncher",  $\phi_s$  increases to its final value of  $-30^\circ$ , while  $m$  increases to  $\sim 2$ . In some designs,  $m$ ,  $\phi_s$  and  $a$  vary while  $B$  remains fixed as well as the bunch length

$$\frac{\beta\lambda}{2\pi} (\Delta\phi_{\text{tot}})$$

and the synchrotron frequency

$$\Omega_0 = \sqrt{\frac{qVA\omega^2 |\sin\phi_s|}{4Mc^2\beta^2}}$$

so that the RF defocusing and the transverse current limit remain constant. (This portion of the RFQ can also be optimized in other ways which maintain adequate current with minimum emittance growth, and these will also be explored.)

The "Accelerator" section follows the "Buncher". In this section  $\phi_s$  and  $m$  are kept constant ( $\phi_s \cong -30^\circ$ ,  $m \cong 2$ ) until the beam energy reaches its final desired value.

The entire RFQ can then be evaluated on its current and emittance properties. The program PARMTEQ is used in this evaluation. This program tracks a set of particles through the RFQ linac, calculating 6-D phase space trajectories with 3-D space charge forces. In the following section we present sample designs developed at LASL and Fermilab for RFQ

projects at BNL, CERN and Fermilab. We also present results of PARMTEQ evaluations of these RFQ's.

### Sample RFQ Designs

The first case we review is "CERN22" designed by Crandall, Stokes and Wangler for the CERN RFQ<sup>6</sup>. This is a 1.38 m long, 202.56 MHz design for accelerating protons ( $H^+$ ) from an energy of 50 keV to 520 keV with a design current of 100 mA. Parameters of this design are shown in Table I and Figure 3.

This design obtains full energy at the end of the "Gentle Buncher" section, and therefore has no separate "Accelerator" section. The design has a relatively large aperture (.68 cm) to accommodate the large space charge (100 mA) with a relatively large emittance ( $E_{rms} = .043 \pi \text{ cm-mr}$ ). The RFQ constructed from this design will be used as the injector for the CERN Linac I.

The second case we consider is "BNL4" designed by Crandall et al. for Brookhaven<sup>7</sup>. This is designed for a very low current ( $\approx 1 \text{ mA}$ ) polarized  $H^-$  beam, and accelerates the beam from 20 keV to 750 keV for the BNL AGS Linac. The major design goal is to minimize emittance dilution.

Figure 4 and Table 2 show the parameters and performance of this system. In low current simulation dilution is  $\sim 10\%$ . The current carrying capacity of this RFQ is somewhat less than the CERN case. Space charge limit formulae give current limits of  $\sim 60 \text{ mA}$ , and simulations obtain reasonably good acceptance at  $\sim 40\text{-}50 \text{ mA}$ .

The third case we consider is a design for the Fermilab preaccelerator, which has design requirements intermediate between the two above cases. The injection energy is 30 keV, final energy is 750 keV, and a beam current  $\gtrsim 30 \text{ mA}$  is desired. We have chosen a peak voltage of  $\sim 60 \text{ kV}$  to place power requirements below 100 kW, and have chosen apertures similar to the BNL case

to minimize phase space dilution and final emittance. The design has good acceptance at 45 mA current, and is shown in Table 3 and Figure 5.

This case is a first suggested design for the Fermilab project, which will be further modified as design requirements become more precise, and current and emittance requirements are balanced.

The length of the RFQ is, however, approximately set by our choice of initial and final energies. The current and emittance requirements set the average aperture (large aperture for large current, small aperture for small emittance). Simulations indicate that RFQ performance is not greatly sensitive to design details within the general design procedure outlined above. We do not expect a final design to be radically different from that suggested here.

#### References

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of the Linac and Accelerator Division in initiating the project.

Table I CERN RFQ Parameters

Frequency	202.56 MHz
Length	138.24 cm
Vane voltage	108.3 kV
Average radius	0.678 cm
Minimum radius	0.421 cm
$B_0$ (focusing parameter)	5.50
Initial energy	50 keV
Final energy	520 keV

PARMTEQ Results ( $\epsilon_{\perp}$  (initial) = .165)

Initial Current (ma)	Final (ma)	Acceptance (%)	$\epsilon_{\perp}^N$ (RMS) (mm-mr)	$\epsilon_L$ (RMS) (keV-R)
0	0	97.8%	0.19	4.70
50	47.2	94.4	0.38	1.57
100	88.3	88.3%	0.44	1.87

Table II Brookhaven RFQ Parameters

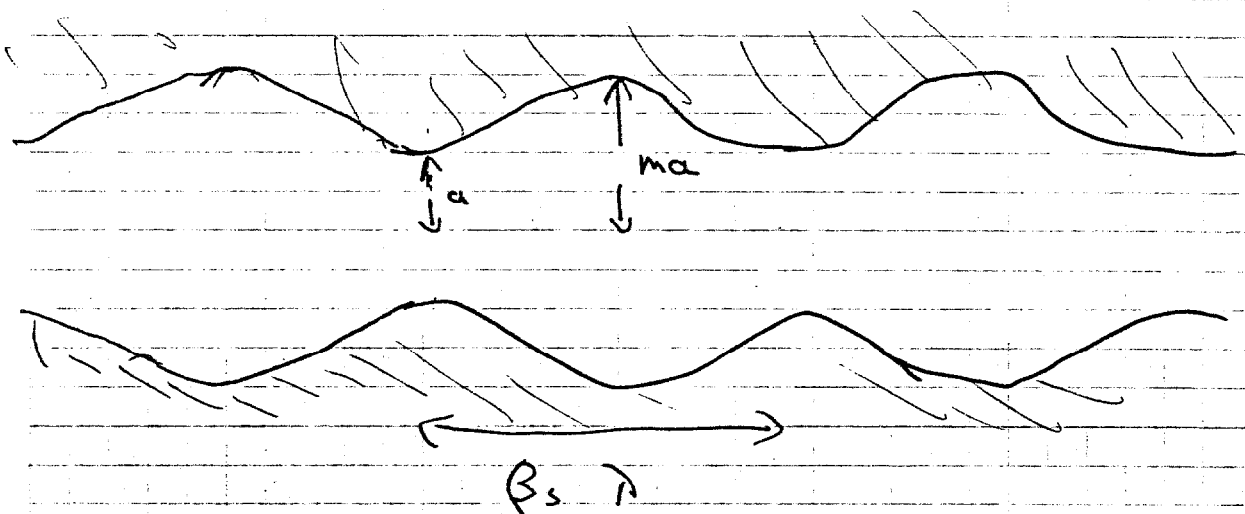
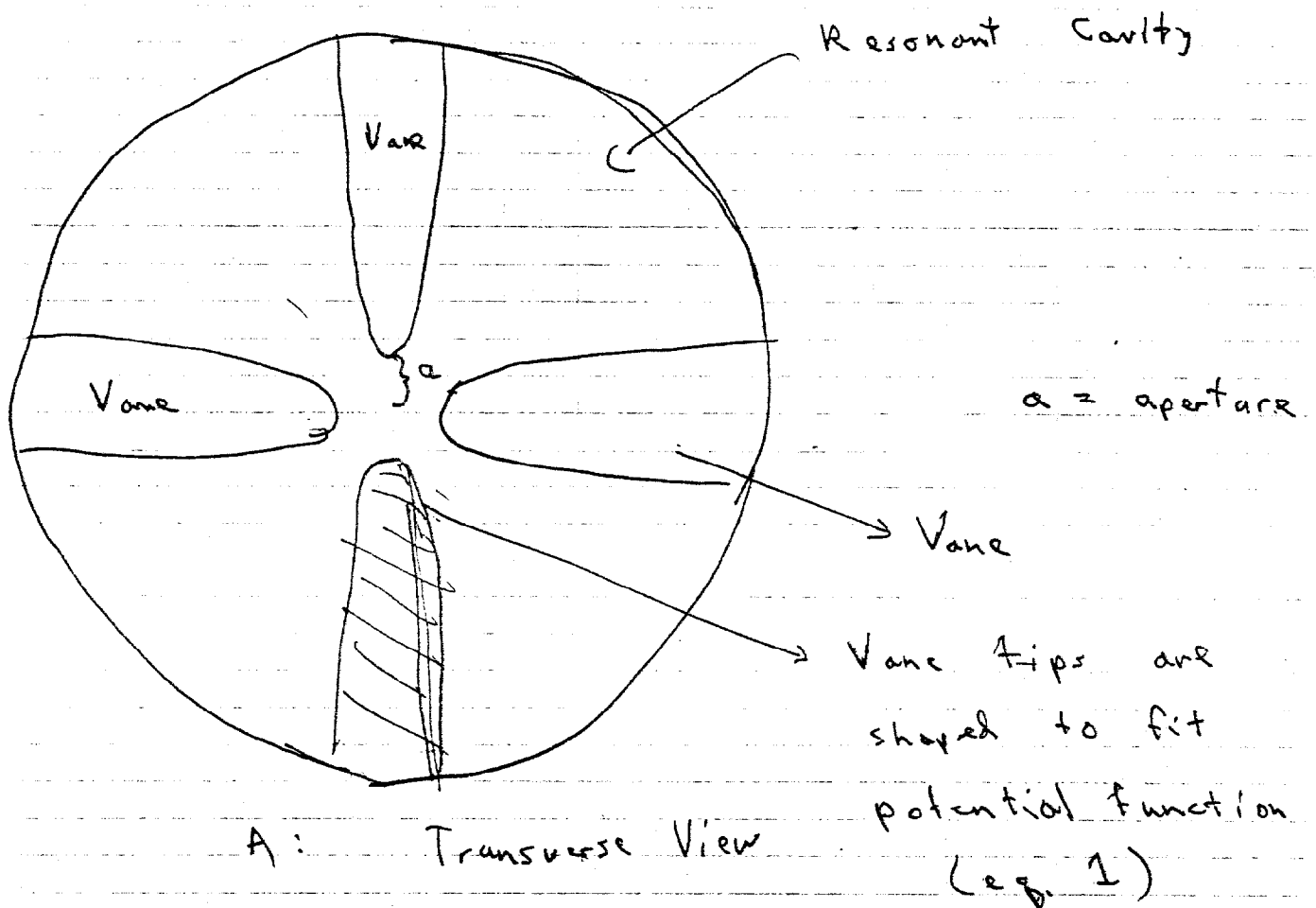
Frequency	201.25 MHz
Length	130.28 cm
Intervane voltage	63 kV
Average radius	0.464 cm
Minimum radius	0.30 cm
Final modulation	1.969
Initial energy	20 keV
Final energy	760 keV
$B_0$ (focusing parameter)	6.96

## Case III Fermilab RFQ Parameters

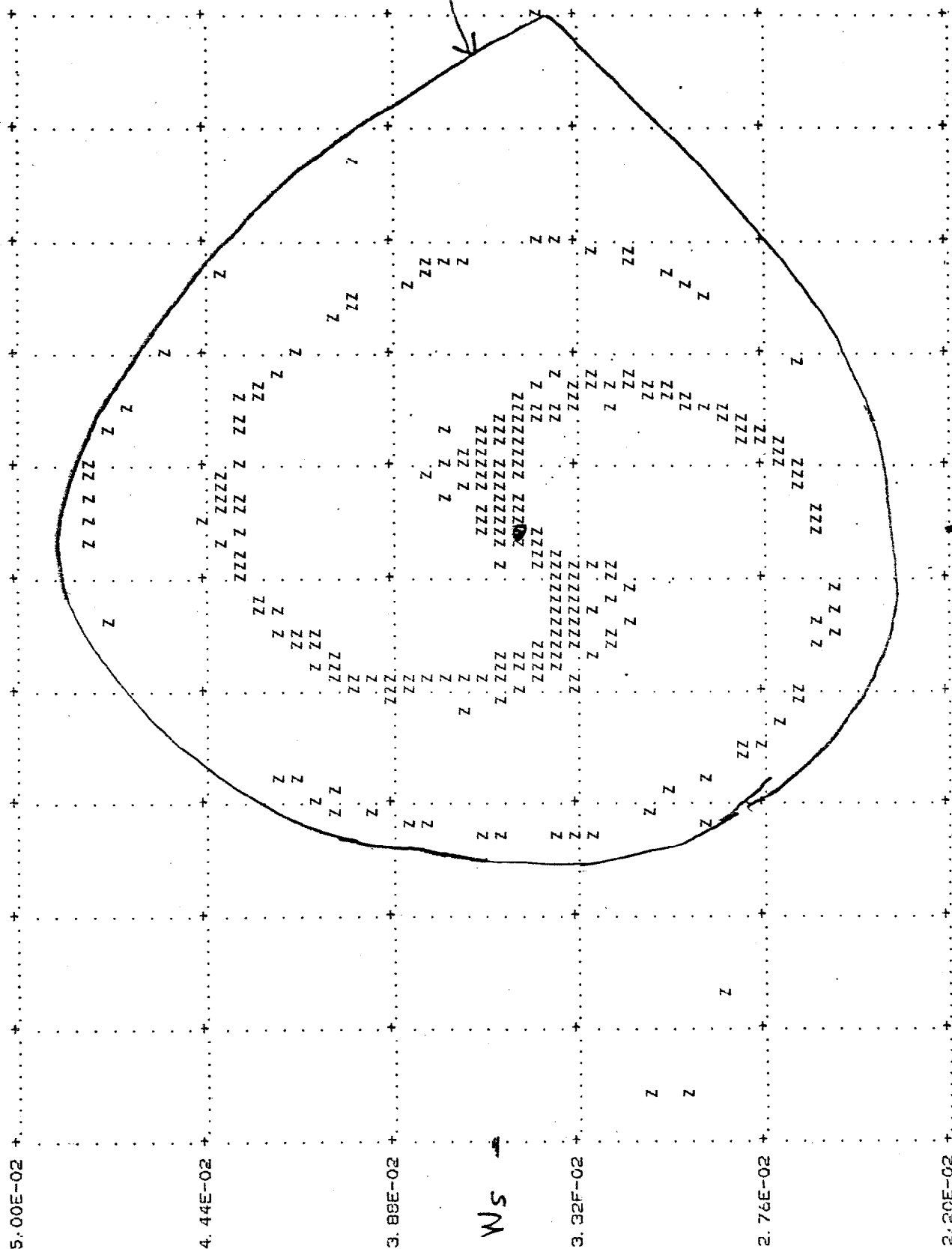
Frequency	200.0 MHz
Length	139.7 cm
Intervane voltage	66 kV
Average radius	0.455 cm
Minimum radius	0.285 cm
Final modulation	2.05
Initial energy	20 keV
Final energy	750 keV
Focusing parameter (B)	7.0

PARMTEQ Results ( $\epsilon_{\perp}^{\text{initial}} = 0.125$ )

Initial Current (ma)	Final (ma)	Acceptance (%)	$\epsilon_{\perp}^N$ (RMS) (mm-mr)	$\epsilon_L$ (keV-R)
0	0	99.0	0.14	3.5
15	14.3	95.0	0.17	2.35
30	27.2	90.6	0.205	1.67
60	44.5	74.0%	0.24	1.66



B: Longitudinal view of opposite vane tips, showing modulation of aperture



-3.142 -2.513 -1.885 -1.257 -.628 -.000 .628 1.257 1.885 2.513 3.142

$\phi_s$

Fig. 2.

RF "Bucket" containing beam in RFQ at end of "Shaper"

Figure 3: Parameters of CERN RFP

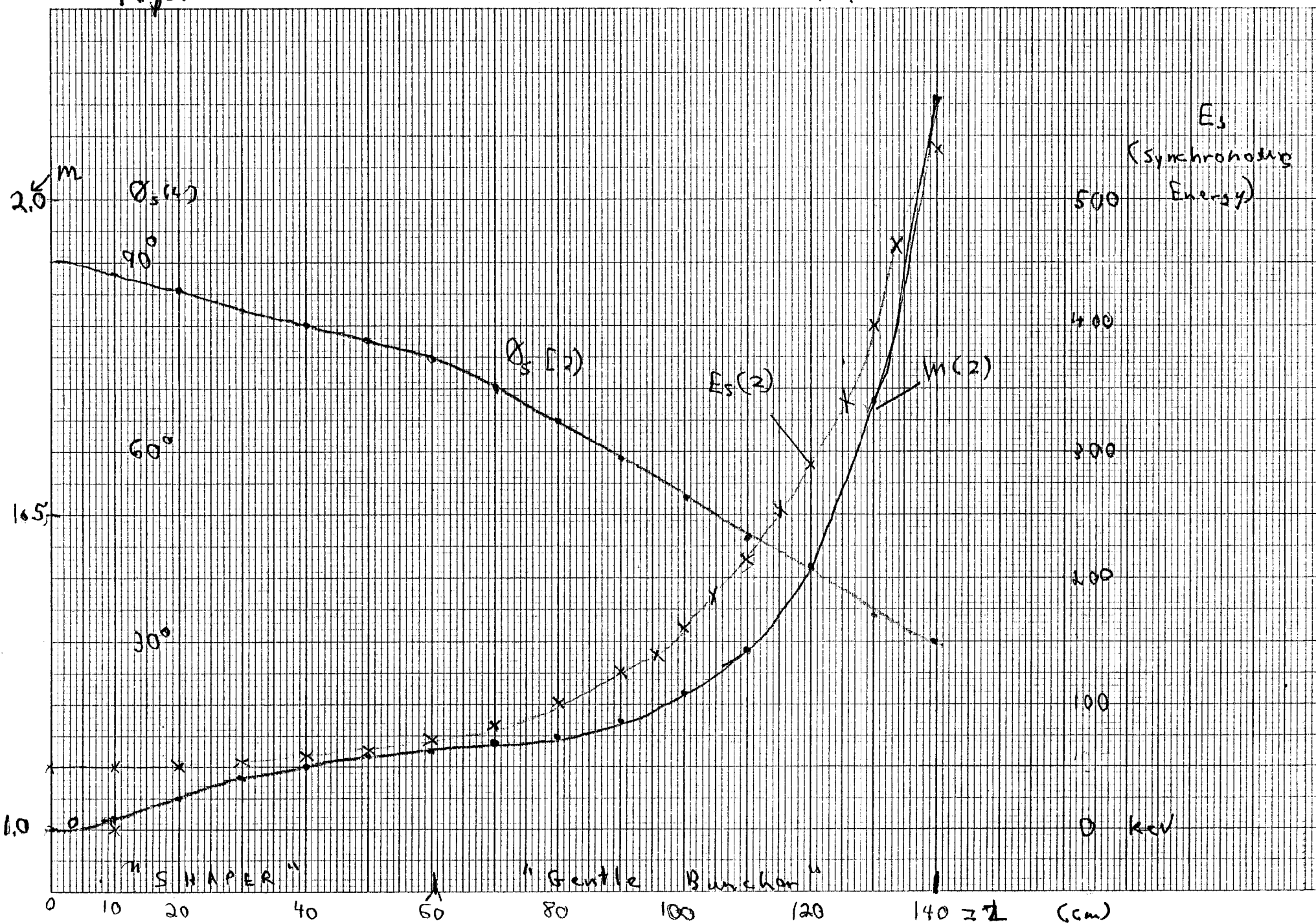
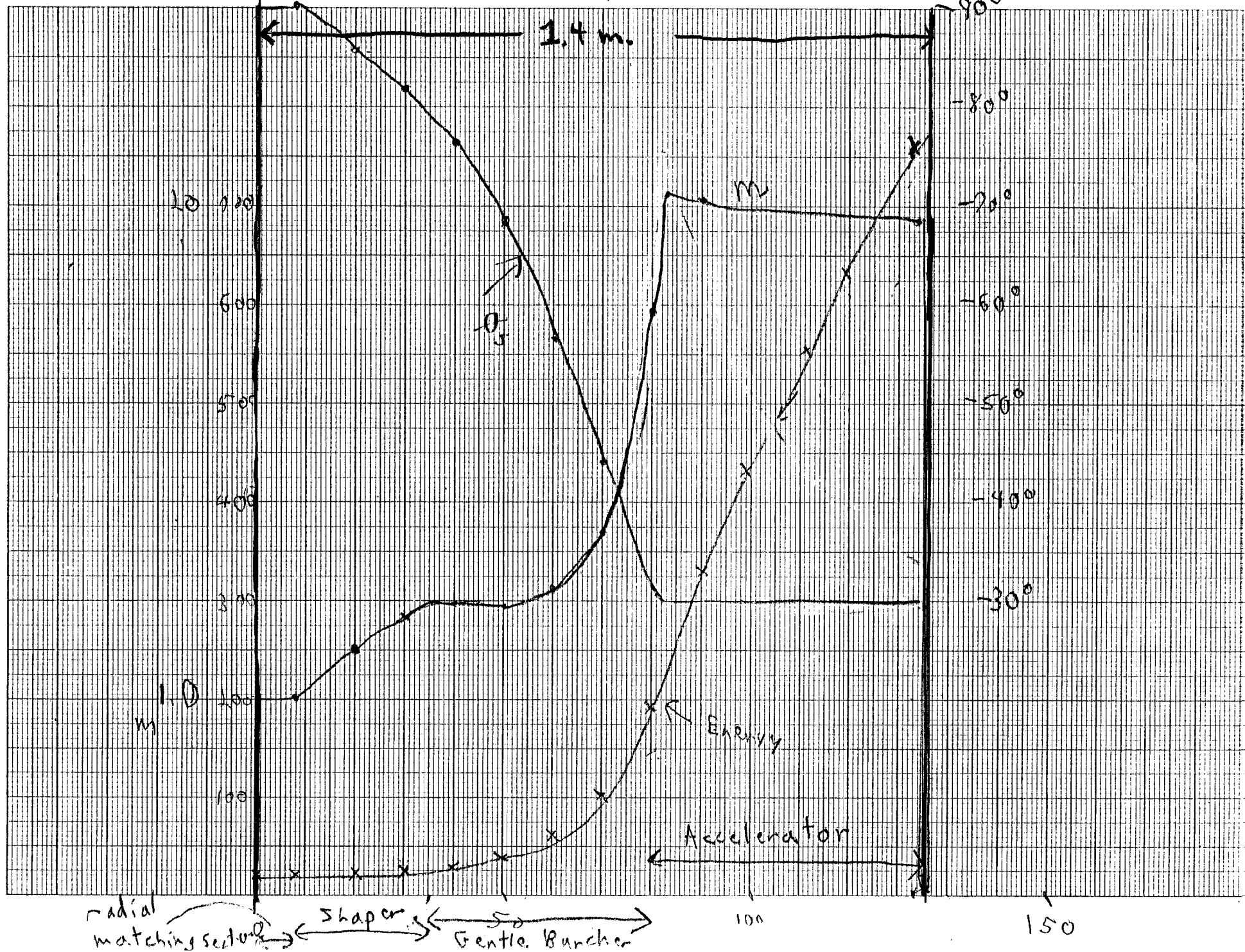


Figure 4: Brookhaven RFQ Figure 4







SUBJECT

Figure 5

Fermilab RFQ Parameters

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